Firm Dynamics in the Neoclassical Growth Model

Omar Licandro
University of Nottingham

2015 RIDGE December Forum
Aims

The Economy

Stationary Equilibrium

Technical Progress

Summary

Ramsey-Hopenhayn Model
Ramsey-Hopenhayn Model

- A continuous time optimal growth model with heterogeneous firms based on Hopenhayn (1992)
  An extension of the Neoclassical growth model
Ramsey-Hopenhayn Model

- A continuous time optimal growth model with heterogeneous firms based on Hopenhayn (1992)
  An extension of the Neoclassical growth model
- Firms’ productivity is heterogeneous; the dynamics of firms is governed by firms’ entry and exit

  **Selection** tends to eliminate low productive firms and reallocate resources towards high productive firms
Ramsey-Hopenhayn Model

- A continuous time optimal growth model with heterogeneous firms based on Hopenhayn (1992)
  
  An extension of the Neoclassical growth model

- Firms’ productivity is heterogeneous; the dynamics of firms is governed by firms’ entry and exit
  
  **Selection** tends to eliminate low productive firms and reallocate resources towards high productive firms

- As a consequence, more selective economies are more productive, produce more and have larger welfare
Ramsey-Hopenhayn Model

- A continuous time optimal growth model with heterogeneous firms based on Hopenhayn (1992)
  An extension of the Neoclassical growth model
- Firms’ productivity is heterogeneous; the dynamics of firms is governed by firms’ entry and exit
  \textbf{Selection} tends to eliminate low productive firms and reallocate resources towards high productive firms
- As a consequence, more selective economies are more productive, produce more and have larger welfare
- The analysis is restricted to steady state
- We study an economy with zero growth (easy to generalize)
The model

Aims

The Economy

Stationary Equilibrium

Technical Progress

Summary

The model

The competitive dynamic general equilibrium model with entry and exit:

- There is only one good.
- A continuum of heterogeneous, competitive firms.
- Firms technology employs capital (fixed factor) and labor.
- Free entry: the value of entry is equal to the investment cost.
- The initial productivity of entrants is random.
- When productivity is too small, firms optimally exit.
- Capital is partially irreversible: its scrap value is \( \theta \in (0, 1) \).

More efficient economies have larger \( \theta \).
The model

Competitive dynamic general equilibrium model with entry and exit
The model

Competitive dynamic general equilibrium model with entry and exit

- There is only one good
The model

Competitive dynamic general equilibrium model with entry and exit

- There is only one good
- A continuum of heterogeneous, competitive firms
The model

Competitive dynamic general equilibrium model with entry and exit

- There is only one good
- A continuum of heterogeneous, competitive firms
- Firms technology employs capital (fixed factor) and labor
The model

Competitive dynamic general equilibrium model with entry and exit

- There is only one good
- A continuum of heterogeneous, competitive firms
- Firms technology employs capital (fixed factor) and labor
- Free **entry**: the value of entry is equal to the investment cost
The model

Competitive dynamic general equilibrium model with entry and exit

- There is only one good
- A continuum of heterogeneous, competitive firms
- Firms technology employs capital (fixed factor) and labor
- Free entry: the value of entry is equal to the investment cost
- The initial productivity of entrants is random
The model

Competitive dynamic general equilibrium model with entry and exit

- There is only one good
- A continuum of heterogeneous, competitive firms
- Firms technology employs capital (fixed factor) and labor
- Free entry: the value of entry is equal to the investment cost
- The initial productivity of entrants is random
- When productivity is too small, firms optimally exit
The model

Competitive dynamic general equilibrium model with entry and exit

- There is only one good
- A continuum of heterogeneous, competitive firms
- Firms technology employs capital (fixed factor) and labor
- Free entry: the value of entry is equal to the investment cost
- The initial productivity of entrants is random
- When productivity is too small, firms optimally exit
- Capital is partially irreversible: its scrap value is \( \theta \in (0, 1) \)
  More efficient economies have larger \( \theta \)
Preferences

- A mass of measure one of identical households
- A representative household offers inelastically one unit of labor
- The Euler equation is
  \[
  \dot{c}_t = \sigma_t \left( r_t - \rho \right)
  \]
  \[
  \rho > 0 \text{ is the discount rate}
  \]
  \[
  \sigma_t > 0 \text{ is the intertemporal elasticity of substitution}
  \]
- At steady state
  \[
  r_t = \frac{\rho}{2}
  \]
Preferences

- A mass of measure one of identical households
Preferences

- A mass of measure one of identical households
- A representative household offers inelastically one unit of labor
Preferences

- A mass of measure one of identical households
- A representative household offers inelastically one unit of labor
- The Euler equation is

\[ \frac{\dot{c}_t}{c_t} = \sigma_t (r_t - \rho) \]

\( \rho > 0 \) is the discount rate
\( \sigma_t > 0 \) is the intertemporal elasticity of substitution
 Preferences

- A mass of measure one of identical households
- A representative household offers inelastically one unit of labor
- The Euler equation is

\[ \frac{\dot{c}_t}{c_t} = \sigma_t (r_t - \rho) \]

\( \rho > 0 \) is the discount rate
\( \sigma_t > 0 \) is the intertemporal elasticity of substitution

- At steady state \( r_t = \rho \)
Technology

A firm is characterized by a firm-specific productivity $z$.

It requires one unit of capital to produce.

The technology of a firm with productivity $z$ is

$$x = A z^{\alpha} \ell^{1-\alpha}$$

where $A > 0$ and $\alpha \in (0, 1)$.

At equilibrium, output $y = y(z)$ and employment $\ell = \ell(z)$. 
Technology

- A firm is characterized by a firm-specific productivity $z$
Technology

- A firm is characterized by a firm-specific productivity $z$
- It requires one unit of capital to produce
Technology

- A firm is characterized by a firm-specific productivity $z$
- It requires one unit of capital to produce
- The technology of a firm with productivity $z$ is

$$x = A z^\alpha \ell^{1-\alpha}$$

$A > 0$ and $\alpha \in (0, 1)$
Technology

- A firm is characterized by a firm-specific productivity $z$
- It requires one unit of capital to produce
- The technology of a firm with productivity $z$ is
  \[ x = A z^\alpha \ell^{1-\alpha} \]
  where $A > 0$ and $\alpha \in (0, 1)$
- At equilibrium, output $y = y(z)$ and employment $\ell = \ell(z)$
Entry and Exit

Endogenous entry and exit

- Entering firms buy (one unit of) capital before entering
- Then, draw productivity $z$ from density $\phi(z)$
- Expected $z$ at entry is one (without any loss of generality)
- If $z$ is too small, the firm immediately exits and recovers $\theta < 1$
- Notation: the marginal firm has productivity $z^*$
  - $z^*$ is endogenously determined at equilibrium

Exogenous exit

- Incumbent firms exit at the rate $\delta > 0$
- In this case, capital fully depreciates
Entry and Exit

- **Endogenous entry and exit**

  - Entering firms buy (one unit of) capital before entering.
  - Then, draw productivity $z$ from density $\phi(z)$.
  - Expected $z$ at entry is one (without any loss of generality).
  - If $z$ is too small, the firm immediately exits and recovers $\theta < 1$.
  - Notation: the marginal firm has productivity $z^*$, which is endogenously determined at equilibrium.

- **Exogenous exit**

  - Incumbent firms exogenously exit at the rate $\delta > 0$.
  - In this case, capital fully depreciates.
Entry and Exit

- **Endogenous entry and exit**
  - Entering firms buy (one unit of) capital before entering
Entry and Exit

**Endogenous entry and exit**

- Entering firms buy (one unit of) capital before entering
- Then, draw productivity $z$ from density $\varphi(z)$
  
  Expected $z$ at entry is one (without any lost of generality)
Entry and Exit

- **Endogenous entry and exit**
  - Entering firms buy (one unit of) capital before entering
  - Then, draw productivity $z$ from density $\varphi(z)$
    Expected $z$ at entry is one (without any lost of generality)
  - If $z$ is too small, the firm immediately exit and recovers $\theta < 1$
Entry and Exit

- **Endogenous entry and exit**
  - Entering firms buy (one unit of) capital before entering
  - Then, draw productivity $z$ from density $\varphi(z)$
    - Expected $z$ at entry is one (without any loss of generality)
  - If $z$ is too small, the firm immediately exit and recovers $\theta < 1$
  - Notation: the marginal firm has productivity $z^*$
    - $z^*$ is endogenously determined at equilibrium

- **Exogenous exit**
  - Incumbent firms exogenously exit at the rate $\delta > 0$
  - In this case, capital fully depreciates
Entry and Exit

• **Endogenous entry and exit**
  
  • Entering firms buy (one unit of) capital before entering
  
  • Then, draw productivity $z$ from density $\varphi(z)$
    
    Expected $z$ at entry is one (without any lost of generality)
  
  • If $z$ is too small, the firm immediately exit and recovers $\theta < 1$
  
  • Notation: the marginal firm has productivity $z^*$
    
    $z^*$ is endogenously determined at equilibrium

• **Exogenous exit**
  
  • Incumbent firms exogenously exit at the rate $\delta > 0$
    
    In this case, capital fully depreciates
Equilibrium Distribution

Since firms with productivity $z < z^*$ immediately exit, the equilibrium distribution is the truncated entry distribution $\phi(z) = \phi(z) \left( 1 - \Phi(z^*) \right)$, where $\Phi(z)$ is the cumulative entry distribution, s.t., $\phi(z) = \Phi'(z)$.

Average productivity:

$$\bar{z} = \int_{z^*}^{\infty} z \phi(z) \, dz > 1$$

Selection makes average productivity larger than expected productivity at entry.
Equilibrium Distribution

Since firms with productivity $z < z^*$ immediately exit
the equilibrium distribution is the truncated entry distribution

$$\phi(z) = \frac{\varphi(z)}{1 - \Phi(z^*)}$$

$\Phi(z)$ is the cumulative entry distribution, s.t., $\varphi(z) = \Phi'(z)$
Equilibrium Distribution

Since firms with productivity $z < z^*$ immediately exit, the equilibrium distribution is the truncated entry distribution

$$
\phi(z) = \frac{\varphi(z)}{1 - \Phi(z^*)}
$$

$\Phi(z)$ is the cumulative entry distribution, s.t., $\varphi(z) = \Phi'(z)$

Average productivity:

$$
\bar{z} = \int_{z^*}^{\infty} z \phi(z) \, d(z) > 1
$$

Selection makes average productivity larger than expected productivity at entry
Firms’ Problem

- Both goods and labor markets are competitive.
- A firm with productivity \( z \geq z^* \) solves
  \[
  \max \ell A z^\alpha \ell^{1-\alpha} - w \ell
  \]
given the wage rate \( w \).
- The optimal labor demand is
  \[
  \ell(z) = (\frac{1-\alpha}{A w})^{\frac{1}{\alpha}} z
  \]
decreasing in wages and increasing in firm’s productivity.
Firms’ Problem

- Both goods and labor markets are competitive
Firms’ Problem

- Both goods and labor markets are competitive
- A firm with productivity $z \geq z^*$ solves

$$\max_{\ell} \ A z^\alpha \ell^{1-\alpha} - w\ell$$

given the wage rate $w$
Firms’ Problem

• Both goods and labor markets are competitive

• A firm with productivity $z \geq z^*$ solves

\[
\max_\ell \ A z^\alpha \ell^{1-\alpha} - w \ell
\]

given the wage rate $w$

• The optimal labor demand is

\[
\ell(z) = \left( \frac{(1 - \alpha)A}{w} \right)^{\frac{1}{\alpha}} z
\]

decreasing in wages and increasing in firm’s productivity
Labor market equilibrium
Labor market equilibrium

• Equilibrium in the labor market

\[ k \int_{z^*}^{\infty} \ell(z) \phi(z) \, dz = 1 \]

\[ k \] is the total number of firms, equal to aggregate capital per capita
Labor market equilibrium

- Equilibrium in the labor market
  \[
  k \int_{z^*}^{\infty} \ell(z) \phi(z) \, dz = 1
  \]
  
  $k$ is the total number of firms, equal to aggregate capital per capita

- Labor market clearing implies
  \[
  w = (1 - \alpha) A (\bar{z}k)^{\alpha}
  \]
  
  marg. prod. of labor
Labor market equilibrium

- Equilibrium in the labor market

\[ k \int_{z^*}^{\infty} \ell(z) \phi(z) \, dz = 1 \]

\( k \) is the total number of firms, equal to aggregate capital per capita

- Labor market clearing implies

\[ w = (1 - \alpha) A (\bar{z}k)^\alpha \]

\( w \) = marginal product of labor

Selection raises productivity and wages
Aggregate Output

• Aggregate output per capita:
  \[ y = k \int_{\infty}^{z^*} A(z)^{\alpha} \ell(z) \left( \frac{1}{1-\alpha} \right) \phi(z) \, dz \] (1)

• Aggregate technology is Neoclassical at equilibrium:
  \[ y = A(\bar{z}k) \alpha \]

Selection makes the economy more efficient in transforming inputs into output.
Aggregate Output

- Aggregate output per capita:

\[ y = k \int_{z^*}^{\infty} A z^\alpha \ell(z)^{1-\alpha} \phi(z) \, dz \quad (1) \]
Aggregated Output

• Aggregate output per capita:

\[ y = k \int_{z^*}^{\infty} A z^\alpha \ell(z)^{1-\alpha} \phi(z) \, dz \quad (1) \]

• Aggregate technology is Neoclassical at equilibrium

\[ y = A (\bar{z}k)^\alpha \]
Aggregate Output

- Aggregate output per capita:

\[ y = k \int_{z^*}^{\infty} A z^\alpha \ell(z)^{1-\alpha} \frac{\phi(z)}{y(z)} \, dz \]  

(1)

- Aggregate technology is Neoclassical at equilibrium

\[ y = A (\bar{z}k)^\alpha \]

Selection makes the economy more efficient in transforming inputs into output
Aims
The Economy
Stationary Equilibrium
Technical Progress
Summary

Aggregate Output: General Case

Assume $x = F(z, \ell)$ where $F(z, \ell)$ is a Neoclassic production function.

Aggregate output per capita is $y = F(\bar{z}^k, 1) = f(\bar{z}^k)$.

Aggregate technology is Neoclassic.
Capital is measured in efficiency units.
Aggregate Output: General Case

- Assume

\[ x = F(z, \ell) \]

where \( F(.) \) is a Neoclassic production function
### Aggregate Output: General Case

- Assume

$$x = F(z, \ell)$$

where $F(.)$ is a Neoclassic production function

- Aggregate output per capita is

$$y = F(\bar{z}k, 1) = f(\bar{z}k)$$

**Aggregate technology is Neoclassic**
Aggregate Output: General Case

- Assume

\[ x = F(z, \ell) \]

where \( F(.) \) is a Neoclassic production function

- Aggregate output per capita is

\[ y = F(\bar{z}k, 1) = f(\bar{z}k) \]

*Aggregate technology is Neoclassic*

Capital is measured in efficiency units
Firm’s Value
Firm’s Value

- Equilibrium profits are linear on $z/\bar{z}$

\[ \pi(z) = \left( \alpha A \bar{z}^\alpha k^{\alpha-1} \right) \frac{z}{\bar{z}} \]

Profits are the return to capital
Firm’s Value

- Equilibrium profits are linear on $z/\bar{z}$

$$\pi(z) = \left( \alpha A \bar{z}^\alpha k^{\alpha-1} \right) \frac{z}{\bar{z}}$$

- Profits are the return to capital

- Firms’ value function at steady state is

$$v(z) = \begin{cases} 
\frac{\pi(z)}{\rho + \delta} & \text{if } z \geq z^* \\
\theta & \text{otherwise}
\end{cases}$$

$v(z)$ is monotonically increasing on $z$
Equilibrium Exit

- The exit condition determines the threshold $z^* \pi(z^*) = \alpha A(\bar{zk})^{\alpha - 1} z^* = \theta (\rho + \delta)$

- A firm exits when $z < z^*$ when the value of staying in the market is smaller than the scrap value of capital $\theta \bar{zk} = \rho + \delta$.
Equilibrium Exit

- The **exit condition** determines the threshold \( z^* \)

\[
\pi(z^*) = \alpha A (\bar{z}k)^{\alpha-1}z^* = \theta(\rho + \delta)
\]
Equilibrium Exit

- The **exit condition** determines the threshold $z^*$

\[
\pi(z^*) = \alpha A (\bar{z}k)^{\alpha-1} z^* = \theta(\rho + \delta)
\]

- A firm exits when $z < z^*$

  *When the value of staying in the market is smaller than the scrap value of capital $\theta$*
Equilibrium Exit

- The **exit condition** determines the threshold $z^*$

$$
\pi(z^*) = \alpha A (\bar{z}k)^{\alpha-1} z^* = \theta(\rho + \delta)
$$

- A firm exits when $z < z^*$

When the value of staying in the market is smaller than the scrap value of capital $\theta$

- Notice that

$$
\frac{z^*}{\theta \bar{z}} = \frac{\rho + \delta}{\alpha A \bar{z}^\alpha k^{\alpha-1}}
$$

(EC)
Equilibrium Entry

• The expected value at entry is equal to the investment cost

\[ \Phi(z^*) \theta + \left(1 - \Phi(z^*)\right) \pi(\bar{z}) / (\rho + \delta) = 1 \]

• Free entry condition becomes

\[ 1 - \Phi(z^*) = 1 - \theta \Phi(z^*) = \rho + \delta \alpha A \bar{z} \alpha k \alpha^{-1} \]

(FE)
Equilibrium Entry

- The expected value at entry is equal to the investment cost

\[ \Phi(z^*) \theta + \left(1 - \Phi(z^*)\right) \pi(\bar{z})/(\rho + \delta) = 1 \]
Equilibrium Entry

- The expected value at entry is equal to the investment cost

\[ \Phi(z^*) \theta + \left(1 - \Phi(z^*)\right) \frac{\pi(\bar{z})}{(\rho + \delta)} = 1 \]

- **Free entry condition** becomes

\[
\frac{1 - \Phi(z^*)}{1 - \theta \Phi(z^*)} = \frac{\rho + \delta}{\alpha A \bar{z}^\alpha k^{\alpha-1}} \quad \text{(FE)}
\]
Equilibrium Cutoff

Combining the exit (EC) and free entry (FE) conditions:

$$z^* \theta = 1 - \Phi(z^*)$$

$$\bar{z} \equiv A(z^*)$$

Remind that $$\bar{z}$$ is increasing in $$z^*$$.
Equilibrium Cutoff

- Combining the exit (EC) and free entry (FE) conditions

\[
\frac{z^*}{\theta} = \frac{1 - \Phi(z^*)}{1 - \theta \Phi(z^*)} \bar{z} \equiv A(z^*)
\]

Remind that \(\bar{z}\) is increasing in \(z^*\)

\[
\bar{z} = \frac{1}{1 - \Phi(z^*)} \int_{z^*}^{\infty} z \varphi(z) dz
\]
Equilibrium Cutoff

Proposition 1: \( z^* \) exists and is unique

Corollary: \( z^* \geq \theta \)

It is always better to close down when productivity is smaller than the scrap value.
Equilibrium Cutoff

- **Proposition 1**: $z^*$ exists and is unique
Equilibrium Cutoff

- **Proposition 1**: \( z^* \) exists and is unique

- **Corollary**: \( z^* \geq \theta \)

It is always better to close down when productivity is smaller than the scrap value
Existence and Unicity

Figure: Existence and unicity of $z^*$
Selection and Capital Irreversibility

Proposition 2: $dz_\ast d\theta > 1$

More reversible capital is, more the economy is selective.

Efficiency:

A reduction in the degree of irreversibility ($\theta$) generates productivity gains through selection ($\bar{z}$).
Selection and Capital Irreversibility

• Proposition 2: \[
\frac{dz^*}{d\theta} > 1
\]
Selection and Capital Irreversibility

• Proposition 2: \( \frac{dz^*}{d\theta} > 1 \)

• More reversible capital is, more the economy is selective
Selection and Capital Irreversibility

- Proposition 2: \( \frac{dz^*}{d\theta} > 1 \)

- More reversible capital is, more the economy is selective

- Efficiency
  - A reduction in the degree of irreversibility (\( \theta \))
  - Generates productivity gains through selection (\( \bar{Z} \))
Selection and Productivity Dispersion

Proposition 3: 
\[ dz^* \sigma > 0 \] 
(\( \sigma \) = variance of the entry distribution) Larger the variance is, more likely is to get a high productivity.
Selection and Productivity Dispertation

- Proposition 3: \( \frac{dz^*}{d\sigma} > 0 \)  
  \((\sigma = \text{variance of the entry distribution})\)

Larger the variance is, more likely is to get a high productivity
Neoclassical Model A

Neoclassical Model A

The Neoclassical Model corresponds to the extreme case of a degenerate entry distribution with zero variance. There is a representative firm with productivity $z = \bar{z} = 1$.

Per capita output is $y = A_k^{\alpha}$.

Wages $w = (1 - \alpha) A_k^{\alpha}$.

Profits are $\pi = \alpha A_k^{\alpha} - 1$.

The free entry condition makes $\rho + \delta = \alpha A_k^{\alpha} - 1$.

Irreversibility plays no role: $(FE) \Rightarrow v(1) > \theta$.

The representative firm never exits.
Neoclassical Model A

- The Neoclassical Model corresponds to the extreme case of a degenerate entry distribution with zero variance.

There is a representative firm with productivity $z = \bar{z} = 1$. 

Neoclassical Model A

- The Neoclassical Model corresponds to the extreme case of a degenerate entry distribution with zero variance.

  There is a **representative firm** with productivity $z = \bar{z} = 1$

- Per capita output is $y = Ak^\alpha$
Neoclassical Model A

- The Neoclassical Model corresponds to the extreme case of a degenerate entry distribution with zero variance.

There is a **representative firm** with productivity \( z = \bar{z} = 1 \)

- Per capita output is \( y = Ak^\alpha \)

- Wages \( w = (1 - \alpha)Ak^\alpha \)
Neoclassical Model A

- The Neoclassical Model corresponds to the extreme case of a degenerate entry distribution with zero variance.

There is a **representative firm** with productivity $z = \bar{z} = 1$

- Per capita output is $y = Ak^\alpha$

- Wages $w = (1 - \alpha)Ak^\alpha$

- Profits are $\pi = \alpha Ak^{\alpha-1}$

  The **free entry condition** makes $\rho + \delta = \alpha Ak^{\alpha-1}$
Neoclassical Model A

- The Neoclassical Model corresponds to the extreme case of a degenerate entry distribution with zero variance.

  There is a **representative firm** with productivity \( z = \bar{z} = 1 \)

- Per capita output is \( y = Ak^\alpha \)

- Wages \( w = (1 - \alpha)Ak^\alpha \)

- Profits are \( \pi = \alpha Ak^{\alpha-1} \)

  The **free entry condition** makes \( \rho + \delta = \alpha Ak^{\alpha-1} \)

- **Irreversibility** plays no role: (FE) \( \Rightarrow \ v(1) > \theta \)

  The representative firm never exits.
Neoclassical Model B

Let us assume the support of the productivity distribution is bounded above by $z_{\text{max}}$.

The Neoclassical model is the limit case of $\theta = 1$.

Selection will make $z^* = \bar{z} = z_{\text{max}}$.

Per capita output $y = A(z_{\text{max}}k)^{\alpha}$.

Wages $w = (1 - \alpha)A(z_{\text{max}}k)^{\alpha}$.

The exit condition makes $\rho + \delta = \alpha A(z_{\text{max}}k)^{\alpha - 1}$.
Neoclassical Model B

- Let us assume the support of the productivity distribution is bounded above by $z_{\text{max}}$
Neoclassical Model B

- Let us assume the support of the productivity distribution is bounded above by $z_{\text{max}}$
- The Neoclassical model is the limit case of $\theta = 1$
**Neoclassical Model B**

- Let us assume the support of the productivity distribution is bounded above by $z_{\text{max}}$

- The Neoclassical model is the limit case of $\theta = 1$

- Selection will make $z^* = \bar{z} = z_{\text{max}}$

\[ y = A(z_{\text{max}}k)^{\alpha} \]

\[ w = (1 - \alpha)A(z_{\text{max}}k)^{\alpha} \]

\[ \rho + \delta = \alpha A(z_{\text{max}}k)^{\alpha - 1} \]
Neoclassical Model B

- Let us assume the support of the productivity distribution is bounded above by $z_{\text{max}}$

- The Neoclassical model is the limit case of $\theta = 1$

- Selection will make $z^* = \bar{z} = z_{\text{max}}$

- Per capita output $y = A(z_{\text{max}}k)^\alpha$
Neoclassical Model B

- Let us assume the support of the productivity distribution is bounded above by $z_{max}$
- The Neoclassical model is the limit case of $\theta = 1$
- Selection will make $z^* = \bar{z} = z_{max}$
- Per capita output $y = A(z_{max}k)^{\alpha}$
- Wages $w = (1 - \alpha)A(z_{max}k)^{\alpha}$
Neoclassical Model B

- Let us assume the support of the productivity distribution is bounded above by $z_{\text{max}}$

- The Neoclassical model is the limit case of $\theta = 1$

- Selection will make $z^* = \bar{z} = z_{\text{max}}$

- Per capita output $y = A(z_{\text{max}}k)^{\alpha}$

- Wages $w = (1 - \alpha)A(z_{\text{max}}k)^{\alpha}$

- The exit condition makes $\rho + \delta = \alpha A(z_{\text{max}}k)^{\alpha-1}$
Equilibrium Capital per capita
Equilibrium Capital per capita

From the entry condition (EC)

\[ \bar{z}k = \left( \frac{\alpha A}{\rho + \delta} \right)^{\frac{1}{1-\alpha}} \left( \frac{z^*}{\theta} \right)^{\frac{1}{1-\alpha}} < \left( \frac{\alpha A z_{\text{max}}}{\rho + \delta} \right)^{\frac{1}{1-\alpha}} \]

Neoclassical Model A  selection  Neoclassical Model B
Equilibrium Capital per capita

From the entry condition (EC)

\[ \bar{z}k = \left( \frac{\alpha A}{\rho + \delta} \right)^{\frac{1}{1-\alpha}} \left( \frac{z^*}{\theta} \right)^{\frac{1}{1-\alpha}} \]

Neoclassical Model A \quad \text{selection}

\[ \left( \frac{\alpha A z_{\text{max}}}{\rho + \delta} \right)^{\frac{1}{1-\alpha}} \]

Neoclassical Model B

- Since \( dz^*/d\theta > 1 \)

More selective economies cumulate more capital
Output

Output per capita is

\[ y = (\bar{z}k)^{\alpha} = (\alpha A^\rho + \delta)^{\alpha} \]

Neoclassical model

More selective economies produce more
Output per capita is

\[ y = (\bar{z}k)^\alpha = \left( \frac{\alpha A}{\rho + \delta} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{z^*}{\theta} \right)^{\frac{\alpha}{1-\alpha}} \]

More selective economies produce more
Consumption and Welfare

\[ c = y - i = \left( (\alpha A \rho + \delta) \alpha - \alpha - \delta (\alpha A \rho + \delta) \right) \]

Neoclassical model

Selection raises per capital consumption and welfare.

Selection is welfare improving.
Consumption and Welfare

- **Selection raises per capital consumption and welfare**

Consumption per capita is

\[ c = y - i = \left( \left( \frac{\alpha A}{\rho + \delta} \right)^{\frac{\alpha}{1-\alpha}} - \delta \left( \frac{\alpha A}{\rho + \delta} \right)^{\frac{1}{1-\alpha}} \right) \left( \frac{z^*}{\theta} \right)^{\frac{\alpha}{1-\alpha}} \]

**Selection is welfare improving**
Technical Progress

The model can be extended to the case of exogenous growth

Assume the productivity of incumbent firms grow at the exogenous rate \( \gamma > 0 \) and the entry distribution \( \Phi_t(z) \) has expected productivity at entry equal to \( e^{\gamma t} \)
Summary

• The Ramsey-Hopenhayn model has the Neoclassical model as a limit case

• Idiosyncratic uncertainty increases the probability of high productivity events to occur, raising selection

• Capital reversibility is good, raising selection too

• More selective economies are more productivity, produce more output and welfare

• More selective economies reallocate labor from less to more productive firms through a higher wage rate