Pseudo-wealth and Consumption Fluctuations*

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Abstract

This paper provides an explanation for situations in which the state variables (like the capital stock and labor supply) describing the economy do not change, but aggregate consumption experiences significant changes. We present a theory of pseudo-wealth – individuals perceived wealth that is derived from heterogeneous beliefs and expectations of gains in a bet. This wealth is divorced from real assets that may exist in society. Heterogeneous beliefs in an economy with a market for bets will imply positive pseudo-wealth. Changes in those beliefs (and more particularly, changes in differences in those beliefs)

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will lead to changes in expected wealth and hence to changes in consumption, implying ex-post intertemporal individual and aggregate consumption misallocations and instabilities.

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1 Introduction

This paper provides a new explanation for an important macro-economic phenomenon: There are many occasions in which the (physical) state variables describing the economy (the level of human and natural capital, the amount of plant and equipment) do not exhibit large changes but the state of the economy, including the levels of consumption of the society, changes dramatically.

This paper puts forward the hypothesis that at least some of this volatility arises from fluctuations in what we call pseudo-wealth – wealth that individuals perceive they have, but which is to some extent divorced from the physical assets that exist in society. We show that there can be sudden changes in the aggregate value of this pseudo-wealth, and that these fluctuations in turn can lead high levels of volatility in aggregate consumption and to ex-post intertemporal consumption misallocations (in the sense of having paths of individual and aggregate consumption that are not as smooth as the individuals and the society wished ex-ante). We show, moreover, that the presence of pseudo-wealth can lead to large increases in the level of debt.

There is a challenge, however, in creating a persuasive theory of pseudo-
wealth. If one assumes expectations are simply arbitrarily given, then a sudden change in expectations (the probability distribution of future states of nature) can obviously give rise to marked changes in the value of wealth. There is some evidence that especially dramatic changes in perceptions occur during times of crisis.\(^1\) The recent US financial crisis is just one more example of this. Prior to the crisis, many financial “experts” believed that the likelihood of a housing bubble was negligible (or at least that its effects would be contained), and presumably they persuaded many others that that was the case. Between 2007 and early 2009 there was a massive change in beliefs. By early 2009, it was hard for anyone to maintain that there had not been a bubble (although some in the U.S. Administration and the Fed seemed to claim otherwise). It is, of course, a challenge to reconcile these beliefs and their evolution over time with any theory of rational expectations.

The problem with this theory is that the task of explaining consumption volatility is too easy. This is a legitimate critique of “animal spirits.” More refined theories try to explain how distributions of beliefs change over time as a function of the new information the economy receives.

In this paper, we explore an explanation that is more tethered and less arbitrary. It is based on two key hypotheses:

1. There can exist large differences in views. Differences in views can exist even when individuals have rational expectations, so long as they have access to different information. All that we require is that the

\(^1\)For instance, as reported in Gluzmann, Guzman, and Howitt (2014), experts’ forecasts on GDP growth exhibit larger changes during times of financial crises. Relatedly, Blanchard and Leigh (2013) show the importance of growth forecast errors for the underestimation of fiscal multipliers.
assumptions that give rise to “common knowledge” – a state of affairs where all individuals agree about the probabilities of different events—are not satisfied.

2. Differences in views, with betting markets, give rise to the creation of pseudo-wealth, with the aggregate expected wealth of market participants exceeding true wealth -i.e., a level of wealth consistent with societal beliefs that are feasible. Each side “expects” to win. Betting markets also lead to more uncertainty. If the positive effect of pseudo-wealth creation on demand is larger than the negative effect of uncertainty (due to the increase in precautionary savings) on demand, the result will be an increase in current levels of consumption. It is inevitable that (later) someone’s expectations will be disappointed—indeed, in any betting market, someone is disappointed. The point here is that the disappearance of the bet will lead to destruction of pseudo-wealth. If the pseudo-wealth component was significant, then this moment will have macroeconomic significance.

We present a model of two agents who disagree in the probability that a sunspot event occurs.\textsuperscript{2} There is a market for short-term bets, and given the disagreement of beliefs, in equilibrium both agents will trade a bet. They both believe they have a larger chance of winning, hence they both feel wealthier. However, this cannot be true for the aggregate, as the bet is not creating any real wealth. Once the sunspot occurs, it cannot occur again;

\textsuperscript{2}We focus on those events because we want to isolate the effects of pseudo-wealth changes from those of other changes. In practice, many bets are about matters of economic substance – such as weather there is a housing or oil price bubble.
hence the bet disappears, and even though no real wealth is destroyed, the expected wealth of both agents discontinuously decreases.\textsuperscript{3} \textsuperscript{4}

We obtain two main results in terms of the dynamics of consumption. Firstly, in every period that the sunspot does not occur, the agent that is more optimistic about the realization of this event will experience a decrease of her expected wealth, as the future will look exactly the same as one period before with the difference that the agent will have borrowed to smooth out consumption over time. The opposite will occur for the agent who is more pessimistic about that state of the world, who will win the bet and still will expect the same tree of possibilities for the future.\textsuperscript{5} The implication is that as long as the bet does not disappear, we will observe a decreasing (increasing) path of consumption for the agent who is more optimistic (pessimistic) about the probability of occurrence of the sunspot. This non-smooth path of the individuals’ consumption would not have occurred in a world with common beliefs.

Secondly, the interaction of disagreement of beliefs with a market for bets will create excessive ex-post aggregate consumption volatility, excessive either with respect to a world of common beliefs or to a world with no market for bets. At the moment in which the bet disappears, the agent betting in

\textsuperscript{3}The sunspot can be taken as a metaphor for an event that rarely occurs, like a structural transformation, over which there is not a long history to have properly learned the true probability distribution that governs it. For events of that nature, assuming rational expectations does not seem to be rational.

\textsuperscript{4}One can extend the model to situations where new betting opportunities may open up. The central point is that aggregate consumption will be related to the magnitude of betting opportunities (itself related to the magnitude of differences in judgments).

\textsuperscript{5}This analysis assumes that beliefs are not revised as a function of realization of states. But in this framework there is truly nothing to learn from the realization of states. Matters would be different if the sunspot could be a recurrent state instead of a one-time event.
favor of the sunspot will experience an increase in wealth, and the other agent will experience a decrease in wealth. However, the “pseudo-wealth” component of expected wealth will vanish, as the difference in views that was leading to a perception of higher wealth at the individual level will no longer be relevant. At this moment, aggregate consumption will decrease discontinuously.

If the planner had prohibited the bet, the society would have experienced a smoother path of consumption, and each individual’s consumption profile would have been smooth. However, given individual beliefs, the society would not have been better off ex-ante from the viewpoint of standard Pareto efficiency. But if we allow the planner to take a stance on beliefs, and the planner uses beliefs that are consistent, then the betting equilibrium would be Pareto inferior.

The rest of this section frames our paper in the existing literature. In section 2, we present a simple framework that displays the presence of positive pseudo-wealth. Section 3 solves the model for the certainty equivalence case. Our goal in that section is to illustrate the effects of the dynamics of pseudo-wealth on the dynamics of consumption in a tractable setting. Sections 4 and 5 summarize the main results of the model in terms of individual and aggregate consumption volatility. Section 6 presents the conclusions.

1.1 Related literature

The issue of “excessive” consumption volatility has received much attention in the macroeconomics literature. The term “excessive” indicates that the actual consumption volatility cannot be explained by a benchmark model that
would imply a more stable path of consumption relative to output. Since Friedman, one of the central hypotheses of modern macroeconomics is that agents smooth consumption. The benchmark model typically invoked features a representative agent model with rational expectations and transitory shocks to output. The existing literature offers different types of deviations from that benchmark to explain the higher levels of consumption volatility observed in times of high output volatility.

Aguiar and Gopinath (2007) introduce trend shocks in a real business cycle framework. The volatility of trend shocks is larger in emerging economies than in advanced economies, which implies the higher consumption volatility observed in the former set of economies. But this approach does not really solve the quandary noted above. Unlike our approach, this approach requires large changes in the state variables (represented as trend shocks) to explain large changes in consumption.\(^6\)

Another branch of related literature provides an explanation for changes in current behavior as a response to today’s changes on expectations concerning the evolution of state variables in the future. For example, Beaudry and Portier (2004, 2006) and Jaimovich and Rebelo (2008) present a class of models where news about future total factor productivity drive changes in individuals’ decisions that could lead to a downturn in the present. Relatedly, Lorenzoni (2009) presents a theory of “news shocks”, in which business cycles are driven by changes in the expectations of the individuals about

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\(^6\)Besides, this approach has been criticized by Garcia Cicco, Pancrazzi, and Uribe (2010). They perform an econometric estimation (using long time-series) of the parameters that govern the productivity processes, and obtain a lower variance and persistence of the permanent component of growth shocks, making the model incapable of explaining the high volatility of consumption relative to output in those economies.
the present state of the economy—but expectations are formed on the basis of noisy public sources of information regarding long-run shifts in aggregate productivity. Although in this family of models it is possible to have situations in which the state of the economy changes with no changes in the current state variables, these frameworks—unlike our framework—still rely on changes in the future state variables that are anticipated in the present for explaining changes in the state of the economy observed also in the present.

The literature on learning as the basis of formation of expectations introduces deviations from the full information rational expectations hypothesis (cf. Evans and Honkapohja (2001)). These deviations lead to larger volatility of expected wealth due to the possibility of revisions of expectations. Accordingly, these models lead to larger consumption volatility (for example, Boz, Bora Durdu, and Daude (2011)); Guzman (2014); Heymann and Sanguinetti (1998)). Both this paper and the learning literature are predicated on imperfect knowledge. In both, changes in beliefs have real macroeconomic effects and can lead to volatility. While in the learning literature, macroeconomic fluctuations are related to changes in average beliefs that have macroeconomic consequences, here, fluctuations can arise even if there are no changes in average beliefs: it is changes in the dispersion of beliefs which drives changes in aggregate consumption, and these changes can be triggered in a variety of ways.

There is a large literature that analyzes the consequences of heterogeneous beliefs. Geanakoplos (2010) offers an approach for explaining excessive volatility of asset prices based on the interaction between heterogeneous expectations, collateral constraints, and leverage. Bad news in the economic
environment can be amplified through the interaction between leverage and collateral constraints, leading to large changes in the “marginal buyer”\textsuperscript{7} of an asset, and thus in asset market prices. In this approach, not everyone’s expected wealth is reduced after the shock. (Only the expected wealth of the optimists who owned assets decreases.) In our approach, both optimists and pessimists will suffer a decrease in expected pseudo-wealth after the shock, as the betting market had previously allowed them to exploit differences in beliefs in a way that all of them were feeling too “optimistic” about their future wealth. Scheinkman and Xiong (2003) show how speculative behavior–defined as the agent’s willingness to pay a price for an asset above her valuation of it, due to the belief that he will be able to sell it at an even higher price in the future–in a context of short-sale constraints and overconfidence–defined as the belief of an agent that her information is more accurate than it is–creates asset prices bubbles. In our framework, agents do not speculate –they simply make betting and consumption decisions that they believe are optimal, independent of the potential behavior of others.

More generally, this paper is the first step of a research agenda outlined in Stiglitz (2013, 2014) and advanced in Guzman and Stiglitz (2015) that intends to offer a general framework for understanding situations in which large changes in macroeconomic behavior are observed with no counterpart in the size of changes of the state variables describing the economy.

\textsuperscript{7}The marginal buyer, who is the least optimist of the agents who buy the asset, is a more pessimistic agent after a bad shock.
2 A baseline model of pseudo-wealth

2.1 Basic assumptions

We assume a small open economy with two infinitely lived representative agents, indexed by \( i = A, B \).

In every period, each agent receives an exogenous endowment that we assume it is constant over time. Let \( y^i \) be the endowment that the agents \( i \) receives in each period.

There is a Poisson probability \( \lambda \) for the arrival of a one-time exogenous event, a sunspot. There is disagreement on the value of \( \lambda \): agent \( A \) believes that the sunspot is more likely to occur than agent \( B \) does, i.e., \( \lambda^A > \lambda^B \).

We assume there is a market for short-term bets. Given the presence of disagreement on \( \lambda \), agents will trade a bet in equilibrium.

Agents can borrow in the international credit market at the risk-free interest rate \( r \): We rule out the possibility of default by assumption.

We assume the instantaneous utility function \( u(c^i_t) \) is continuous and strictly concave, \( u'(c^i_t) > 0, u''(c^i_t) < 0 \), where \( c^i_t \) is the level of consumption of individual \( i \) in period \( t \). Besides, we will assume that the utility function belongs to a class of functions for which the positive effect of the creation of pseudo-wealth on the demand for goods dominates the negative effect that comes for the increase in precautionary savings due to the larger risk implied by betting markets. We are interested in analyzing a situation in which a larger expected wealth leads agents to consume more in the present.\(^8\)

\(^8\)For example, this will be true for a CRRA with a risk coefficient smaller than one. This is also trivially true for the quadratic utility function, as it implies no precautionary savings.
2.2 States

Let $\mathcal{S}_t$ be the set of states in period $t$, $s_t \in \mathcal{S}_t$. Until the sunspot occurs, there are two possible states: no sunspot, $s_t = O$, or sunspot, $s_t = S$. Once the sunspot occurs, it cannot occur again, and the state will be $s_t = O$ forever. The sunspot can then be interpreted as a shock to prior beliefs—a shock that vanishes any difference in agents’ priors.

Figure 1 depicts the tree of possible states from period $t$, assuming that there was no sunspot yet.

Figure 1: Space of states

![Tree diagram](image)
2.2.1 Bets and pseudo-wealth

Differences in beliefs give rise to bets. Bets have two effects:

1. Given the disagreement on the true value of \( \lambda \), bets will create pseudo-wealth. Because of the bet, each party believes that it is wealthier. Then, the perceived aggregate wealth exceeds the total “true” wealth. The larger the size of the bet and the larger the discrepancy of beliefs on the probability of occurrence of the sunspot, the larger will be the pseudo-wealth.

2. Bets will create uncertainty about expected wealth. In period \( t \), agents will compute an expected value of wealth \( E_t W^i \) that will positively depend on the size of the bet. However, they will be aware that the bet will create more variance on the value of \( E_{t+1} W^i \), creating more variance in future consumption.

Therefore, every additional dollar of bet will be associated with a marginal benefit that comes from the perceived increase in wealth, and a marginal cost that comes from the increase in the variance of future expected wealth.

In every period the bet gets resolved. One side or the other wins the bet, and the pseudo-wealth that was created gets destroyed. If that were the whole process, pseudo-wealth would be ephemeral, with no real macroeconomic consequences.

If, however, differences in beliefs persist, individuals may once again engage in a bet, and so new pseudo-wealth is created. In the steady state, in every period there is destruction of pseudo-wealth but creation of new pseudo-wealth, as new bets get made.
The bet consists in agent $A$ paying $p_t$ to agent $B$ in period $t$. If $s_t = S$, agent $B$ pays 1 to agent $A$, while if $s_t = O$, agent $B$ pays nothing to agent $A$. Formally, the bet payments $P_t^A$ are described as follows:

\[
P_t^A = \begin{cases} 
1 & \text{if } s_t = S \\
0 & \text{if } s_t = O
\end{cases}
\]  

(1)

and

\[
P_t^B = \begin{cases} 
-1 & \text{if } s_t = S \\
0 & \text{if } s_t = O
\end{cases}
\]  

(2)

The expected pseudo-wealth of agent $A$ in period $t$ is given by

\[
E_t PW_A = \begin{cases} 
E_t \sum_{j=t}^{\infty} \beta (1 - \lambda^A)^{j-t} (\lambda^A - p_j) b_j > 0 & \text{if } s_j = O \ \forall j < t \\
0 & \text{if } \exists j < t : s_j = S
\end{cases}
\]  

(3)

The expected pseudo-wealth of agent $B$ in period $t$ is given by

\[
E_t PW_B = \begin{cases} 
E_t \sum_{j=t}^{\infty} \beta (1 - \lambda^B)^{j-t} (p_j - \lambda^B) b_j > 0 & \text{if } s_{t-1} = O \ \forall j < t \\
0 & \text{if } \exists j < t : s_j = S
\end{cases}
\]  

(4)
where \((\lambda^A - p_j)b_j\) and \((p_j - \lambda^B)b_j\) represent the pseudo-wealth of only one period \(j\) for agents \(A\) and \(B\), respectively. Note that the aggregate pseudo-wealth is strictly positive, and as noted, depends just on the magnitude of the disparity in their beliefs and the size of the bet.

### 2.3 Budget constraints

Let \(d_i^t\) denote debt of agent \(i\) in period \(t\). Agents will face the following budget constraints in each period.

\[
c_t^A + (1 + r)(d_{t-1}^A - P_{t-1}^A b_{t-1}^A) + p_t b_t^A = y + d_t^A \tag{5}
\]

\[
c_t^B + (1 + r)(d_{t-1}^B - P_{t-1}^B b_{t-1}^B) - p_t b_t^B = y + d_t^B \tag{6}
\]

These constraints simply state that each agent’s total expenditure (either in consumption of goods or repayment of debts) and total available endowment (that is either owned or borrowed) must be equal in every period and in every state.

### Aggregates

Adding up the consumption, endowment, and debt of agents \(A\) and \(B\), we define the aggregates:

\[
c_t = c_t^A + c_t^B \tag{7}
\]

\[
y = y^A + y^B \tag{8}
\]

\[
d_t = d_t^A + d_t^B \tag{9}
\]
The set of static aggregate budget constraints is represented by

\[ c_t + (1 + r_{t-1})d_{t-1}' = y_t + d_t' \quad \forall t \]  \hspace{1cm} (10)

Finally, we define aggregate expected wealth and pseudo-wealth:

\[ E_t W = E_t W^A + E_t W^B \]  \hspace{1cm} (11)

\[ E_t PW = E_t PW^A + E_t PW^B \]  \hspace{1cm} (12)

2.4 Optimization

Consumers are forward-looking. In period \( t \), each agent chooses the sequence of consumption and bets in order to maximize the expected present value of utility, subject to the budget constraints:

\[ \max_{\{c^i_j, b^i_j\}_{j=t}^\infty} E_t \sum_{j=t}^\infty \beta^{j-t} u(c^i_j), \quad i = A, B \]  \hspace{1cm} (13)

where \( \beta \in (0, 1) \) is the discount factor, identical for all agents.

Every time the no sunspot state occurs, there will be a winner and a loser of the bet but the future will look exactly as it looked one period before. Then, one agent will be wealthier and the other less wealthy, but they will both have same expectations for the future that they had the period before. That is, the realizations of states act as wealth shocks, which implies a need to reoptimize in each period. The next section offers the solution of the model for the case of preferences defined over a quadratic utility function.
3 The certainty equivalence case

In this section we analyze the effects of pseudo-wealth on consumption volatility in the most tractable setting that allows us to obtain straightforward closed-form solutions, isolating the effects of precautionary savings. We assume that preferences take the form of a quadratic utility function. Our point is to describe how disagreement can lead to changes in macroeconomic behavior through wealth effects, independently of other effects, and this simplification of preferences serves our purpose.

We assume both agents’ preferences are defined by a quadratic utility function:

\[ u(c^i_t) = \alpha c^i_t - \gamma c^i_t^2, \quad i = A, B \]  

(14)

Assumption 1

\[ \frac{\alpha}{2\gamma} > \max \left\{ \frac{y^A}{1 - \beta} + \frac{\bar{b}(\lambda^A - \lambda^B)}{1 - \beta(1 - \lambda^A)}, \frac{y^B}{1 - \beta} + \frac{\bar{b}(\lambda^A - \lambda^B)}{1 - \beta(1 - \lambda^B)} \right\} \]

where \( \bar{b} \) is the maximum bet that can be reached in equilibrium, which is a fixed point that solves \( b_t^A = b_t^B \) as a function of the difference between between the private beliefs and the bet price (whose maximum value is \( \lambda^A - \lambda^B \)) and the expected wealth, which in turn depends on the equilibrium size of the bet. Assumption 1 assures that the marginal utility of consumption is always positive.

We also assume \( \beta(1 + r) = 1 \), which implies that agents want to have a perfectly smooth path of (expected) consumption over time.
Expected wealth  The expected wealth of each agent in period $t$, $E_t W^i$, will be composed of three parts. Firstly, the expected value of the endowment the agent receives. Secondly, due to the difference between the expected probability of occurrence of a sunspot and the price of the bet, each agent feels wealthier: this is what we call pseudo-wealth, which will also be an expected value. Finally, we must subtract the debt payments that must be paid in period $t$. Then,

$$E_t W^i = \frac{y^A}{1 - \beta} + E_t PW^i - (1 + r)d_{t-1}^{s_{t-1},i}$$  \hspace{1cm} (15)

Intertemporal budget constraints  With a quadratic utility function, agent $i$ faces the following intertemporal budget constraint:

$$\sum_{j=t}^{\infty} \beta^j E_t (c^i_j) = \frac{Y^i}{1 - \beta} + E_t PW^i - (1 + r)d_{t-1}^{s_{t-1},i}$$  \hspace{1cm} (16)

3.1 Decisions

Timing of decisions  At the beginning of period $t$, $\{c^i_t, d^i_t, b^i_t\}$ are decided. At the end of the period, the state is realized, and the payoffs of the bet are added to $d^i_t$ to determine $d^{s_{t},i}$ (see table 1).

Bets  When deciding how much to bet, agents face a trade-off. One additional dollar of bet in period $t$ will generate a positive effect on expected wealth in period $t$, because bets create pseudo-wealth. However, one more dollar of bets will also increase the variance of expected wealth in period $t + 1$. 

Each agent will choose $b_i^t$ in order to maximize the value of expected utility. In the case of quadratic utility function, the expected value of utility in period $t$ is given by

$$EU_i^t \equiv EU(c_i^t) = \alpha(1 - \beta)E_tW^i - \gamma(1 - \beta)^2E_tW^{i^2}$$  \hspace{1cm} (17)$$

which can also be expressed as

$$EU_i^t = \alpha(1 - \beta)E_t^i - \gamma(1 - \beta)^2[Var(W_i^t) + (E_i^tW_i^t)^2]$$ \hspace{1cm} (18)$$

where Var denotes variance. Then, $b_i^t$ is the solution to

$$\sum_{j=t}^{\infty} \beta^{j-t} \frac{dEU_j^i}{db_i^t} = \sum_{j=t}^{\infty} \beta^{j-t} \frac{\partial EU_j^i}{\partial E_i^tW_i^t} \frac{\partial E_i^tW_i^t}{\partial b_i^t} + \sum_{j=t+1}^{\infty} \beta^{j-t} \frac{\partial EU_j^i}{\partial Var(W_j^i)} \frac{\partial Var(W_j^i)}{\partial b_i^t} = 0$$ \hspace{1cm} (19)$$

Then,

$$b_i^A \equiv b_i^A(p_t) = \frac{(\lambda^A - p_t)\{\alpha - 2\gamma(1 - \beta)[\frac{y^A}{1-\beta} + E_t \sum_{j=t+1}^{\infty} \beta(1 - \lambda^A)]^{j-t}(\lambda^A - p_j)b_j - (1 + r)d^{st.\lambda^A}_t\}}{2\gamma(1 - \beta)[(\lambda^A - p_t)^2 + \beta\lambda^A(1 - \lambda^A)]}$$ \hspace{1cm} (20)$$

and

$$b_i^B \equiv b_i^B(p_t) = \frac{(p_t - \lambda^B)\{\alpha - 2\gamma(1 - \beta)[\frac{y^B}{1-\beta} + E_t \sum_{j=t+1}^{\infty} \beta(1 - \lambda^B)]^{j-t}(p_t - \lambda^B)b_j - (1 + r)d^{st.\lambda^B}_t\}}{2\gamma(1 - \beta)[(p_t - \lambda^B)^2 + \beta\lambda^W(1 - \lambda^W)]}$$ \hspace{1cm} (21)$$

The equilibrium price $p_t^*$ solves

$$b_i^A(p_t^*) = b_i^B(p_t^*)$$ \hspace{1cm} (22)$$
The following two assumptions assure that agent $A$’s demand for bets is decreasing in $p_t$ and agent $B$’s supply of bets is increasing in $p_t$:

**Assumption 2**

$$\beta \lambda^W (1 - \lambda^W) > (\lambda^W - p_t)^2$$

**Assumption 3**

$$\beta \lambda^B (1 - \lambda^B) > (p_t - \lambda^B)^2$$

**Lemma 1** Under assumptions 2 and 3, if $s_t = O$, then $p_{t+1} > p_t$.\(^9\)

**Proof of Lemma 1** If $s_t = O$, then $A$ adds $p_t b_t$ to $d_{t+1}^A$ and $B$ subtracts $p_t b_t$ to $d_{t+1}^B$. From (20) and (21), $\frac{\partial b_i}{\partial d_{t+1}^{st,i}} > 0$. Then, $b_t^A$ increases and $b_t^B$ decreases. Assumptions 2 and 3 guarantee that the slopes of the demand and supply curves imply a higher price for the bet in the new equilibrium. QED

**Individual consumption and borrowing** Each individual maximizes expected utility subject to (16). In this case of quadratic utility function, the general solution is given by

$$c_t^i = y^i + (1 - \beta) [E_t PW^i - (1 + r)d_{t-1}^{s_{t-1,i}}] \quad (23)$$

\(^9\)This is a result that comes from the quadratic utility function assumption. It does not necessarily arise with a general utility function, but the results obtained below would still hold under a more general representation.
where debt in period $t$ for agents $A$ and $B$ are given by the following expressions (assuming that initial debt in $t = 0$ is zero for each individual):

$$d_t^A = \begin{cases} 
\sum_{j=0}^{t}[(1 - \beta)E_jPW^A + p_jb_j] & \text{if } s_t = O \\
\sum_{j=0}^{t}[(1 - \beta)E_jPW^A + p_jb_j] - b_t & \text{if } s_t = S \lor s_t = O'
\end{cases}$$  \hspace{1cm} (24)

$$d_t^B = \begin{cases} 
\sum_{j=0}^{t}[(1 - \beta)E_jPW^B - p_jb_j] & \text{if } s_t = O \\
\sum_{j=0}^{t}[(1 - \beta)E_jPW^B - p_jb_j] + b_t & \text{if } s_t = S \lor s_t = O'
\end{cases}$$  \hspace{1cm} (25)

### Aggregate consumption and debt

Aggregate consumption is governed by the following expression:

$$c_t = y + (1 - \beta)[E_tPW - (1 + r)d_{t-1}]$$  \hspace{1cm} (26)

Aggregate debt is given by

$$d_t = (1 - \beta)\sum_{j=0}^{t}E_jPW$$  \hspace{1cm} (27)

As long as the sunspot does not occur, aggregate debt will increase over time.
4 Pseudo-wealth and individual consumption volatility

This section presents a set of lemmas and propositions that characterize the evolution of individuals’ consumption.

When the sunspot occurs, pseudo-wealth disappears for both agents. Therefore, it is not possible to have a simultaneous increase in $E_t W^A$ and $E_t W^B$ if $s_t = S$. The following lemma shows that such a simultaneous increase in expected wealth is also impossible if $s_t = O$ (we use the notation $\Delta X = X_t - X_{t-1}$).

**Lemma 2** Under assumptions 2 and 3, it is not possible to have simultaneously $\Delta E_t W^A > 0$ and $\Delta E_t W^B > 0$.

**Proof of Lemma 2** If $s_t = S$, $E_t PW$ is reduced to zero, hence $\Delta E_t W < 0$. Suppose (by contradiction) $s_t = O$ and $\Delta E_t W^A > 0$ and $\Delta E_t W^B > 0$. Then, due to (20) and (21), we must have $\Delta b_t < 0$. But $\Delta E_t PW = (\lambda^A - \lambda^B)\Delta b_t$, hence $\Delta E_t W < 0$, a contradiction. QED

**Proposition 1** Under assumptions 2 and 3, if $s_t = O$ then $c^A_{t+1} < c^A_t$.

**Proof of Proposition 1** From lemma 1, $\Delta p_{t+1} > 0$. Therefore, $\lambda^A - p_t > \lambda^A - p_{t+1}$ and $p_t - \lambda^B < p_{t+1} - \lambda^B$. If $\Delta b_{t+1} < 0$, $\Delta E_{t+1} W^A < 0$ and the proposition would follow. If $\Delta b_{t+1} > 0$, then $\Delta E_{t+1} W^B > 0$. From lemma 2, $\Delta E_{t+1} W^A < 0$, what concludes the proposition. QED

**Proposition 2** Under assumptions 2 and 3, if $s_t = O$ then $c^B_{t+1} > c^B_t$. 

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Proof of Proposition 2: From lemma 1, $\Delta p_{t+1} > 0$. Therefore, $\lambda^A - p_t > \lambda^A - p_{t+1}$ and $p_t - \lambda^B < p_{t+1} - \lambda^B$. If $\Delta b_{t+1} > 0$, then $\Delta E_{t+1}^{W^B} > 0$. From lemma 2, $\Delta E_{t+1}^{W^A} < 0$, and the proposition would follow.

Suppose that $\Delta b_{t+1} < 0$. Let $a^0 = (b^0, p^0)$ be the initial equilibrium, before the shock. Let $a^1 = (b^1, p^1)$ be the equilibrium after the shock if both the elasticity of supply of bonds with respect to $p$ and $d^{s.t.B}$ were zero. Let $a^2 = (b^2, p^2)$ be the equilibrium if the elasticity of supply of bonds with respect to $p$ is the actual one defined by (21), with $b^1 = b^0$ and $p^1 > p^0$. Let $a^3 = (b^3, p^3)$ be the new equilibrium after the realization of the state, with $b^3 < b^0$ and $p^3 > p^0$. Let $EU^B(a^2)$ be the expected utility of agent $B$ when the vector of bets and prices is $a^2$. Then, under assumption 1, $EU^B(a^1) > EU^B(a^0)$, because there would increase in wealth with no increase in variance. Also, $EU^B(a^2) > EU^B(a^1)$, because $a^2$ is obtained from the best response function (21) to $p$ while $a^1$ is not. Also, $EU^B(a^3) > EU^B(a^2)$, because $a^3$ is the best response to $\Delta d_t^{Q.B}$ while $a^2$ is not. Therefore, $EU^B(a^3) > EU^B(a^0)$. Suppose (by contradiction) $E_t W^B(a^3) \leq E_t W^B(a^0)$. Then, by continuity of the function $EU^B(E_t^W)$, $B$ could have obtained more utility by reducing expected wealth and variance of wealth, a contradiction. Therefore, $E_t W^B(a^3) > E_t W^B(a^0)$, which concludes the proof. QED

Discussion of propositions 1 and 2. The results obtained in these propositions resemble the implications of hyperbolic discounting. Both agents rationally want to smooth out consumption over time, but they cannot achieve that outcome. Agent $A$ believes she is richer than what she would be if she only received the endowment $y^A$ due to the existence of the bet and
her subjective beliefs. Therefore, she borrows in the first period. But if the sunspot does not occur in that period, she will face the same tree of possibilities ahead as the one she expected at the beginning of the first period, but she will have positive debt to repay. Therefore, she will be less rich, and her consumption will have to diminish. The opposite occurs to agent B, who gets a positive payment from the bet while the sunspot does not occur.

In this setup, realization of states is equivalent to permanent wealth shocks.

5 Pseudo-wealth and aggregate consumption volatility

This section shows the main proposition of our paper, namely, that ex-post consumption volatility increases as the result of creation of pseudo-wealth.

Proposition 3 At $s_t = S$, there is a discontinuous decrease in aggregate consumption

Proof of Proposition 3 In state $O$, aggregate wealth is

$$EW_t/(s_t = O) = \frac{y}{1-\beta} + [E_t PW - (1 + r)d_t' - 1]$$ (28)

while in state $S$ we have

$$EW_t/(s_t = S) = \frac{y}{1-\beta} - (1 + r)d_{t-1}' < EW_t/(s_t = O)$$ (29)

Then, proposition follows from (26). QED
Definition 1 (Stiglitz, 1982). We say that beliefs satisfy group rationality if

\[ \frac{1}{2} \lambda^A + \frac{1}{2} \lambda^B = \lambda \]  

(30)

where \( \lambda \) is the true probability of occurrence of sunspot.

Corollary 1 Under a utilitarian social welfare function and group rationality, the existence of the betting market decreases expected present value of welfare.

Assessing welfare gains under heterogeneous beliefs is a complicated problem that requires welfare criteria able to deal with beliefs heterogeneity.

The corollary only establishes that prohibiting the bet can increase welfare from the viewpoint of the beliefs of the planner that considers the true probability to be \( \lambda \), defined by (30). But such a prohibition would not increase ex-ante expected utility for agents \( A \) and \( B \), given their beliefs —indeed, both agents would be strictly worse-off ex-ante with such a prohibition given their beliefs.

This corollary is a particular case of the case of “reasonable beliefs” —i.e., beliefs that are a convex combination of agents’ beliefs (see Brunnermeier, Simsek, and Xiong (2014)).

6 Conclusions

This paper has shown that, when there are differences in beliefs, the amounts that betting individuals expect to receive from other individuals may differ markedly from the amounts that these same individuals expect to pay. This
disparity in (the present discounted value of) expected transfer payments we refer to as pseudo-wealth. We have noted that there can be large changes in the aggregate value of pseudo-wealth, and that these changes in aggregate pseudo-wealth can give rise to large fluctuations in consumption.

Our analysis does not need to assume that there is a well-defined distribution of probabilities that it is correct, or that such a distribution is known by the agents of the economy. Our approach is reasonable when agents form beliefs over a one-time event.

While the framework could be interpreted as a deviation from the rational expectations assumption, we could alternatively surmise that the differences in beliefs arise due to differences in the agents’ sets of information, such that each agent either does not know the information the other agent receives, or trusts her own information more (this latter case also being a deviation from the conventional rational expectations assumption).

Indeed, even after the event has occurred, we cannot be sure which of the two individuals was “right.” And, it is absurd to think that all individuals would share the same beliefs about the likelihood of these occurrences, that there would be common knowledge. If agents don’t share the same beliefs, then there is room for a bet that increases pseudo-wealth.

This paper is the first step of a research agenda that intends to offer a general framework for understanding situations in which large changes in macroeconomic behavior are observed together with changes in the state variables describing the economy that are significantly smaller. In an economic downturn, there can even be negative pseudo-wealth. For example, in a borrower-lender relationship, if the borrower is more optimistic than
the lender about the capacity of repayment of borrowings, then the level of
credit in equilibrium will be lower than with common beliefs. The borrower
believes he is paying more than the lender believes he is receiving. In times of
high uncertainty (like recessions) such dispersions in beliefs easily arise, and
contribute to a shrinking of lending, with the obvious macroeconomic conse-
quences. The dynamics of pseudo-wealth–its formation, dissolution, and its
aggregate persistence both positively and negatively–help explain macroeco-
nomic volatility and gives insight into the nature of persistent booms and
busts—a fact largely unexplained by the existing literature.

References


